Resonance Analysis of the Train Slab Track System Considering Moving Load

Kiran Chholak[1], Prof. Nirendra Dev[2]

Delhi Technological University, New Delhi

E-mail- kiran111766@gmail.com , nirendradev@dce.ac.in

Abstract: The dynamic response of the slab track under vehicle activity is examined in the article. The vehicle-slab track coupling model are designed to simulate the interaction at any speed using the finite element method and multibody dynamics. By using mathematical derivations and numerical simulations, the resonance mechanism and conditions of the slab track system are analyzed. Wheel-rail contact force response performance and rail displacement are examined by evaluating the effects of train speed on the slab track. The impact of stiffness of rail pad and bearing stiffness of slab on resonant frequency phenomena of the vehicle track system is evaluated. Increasing the stiffness of rail pad will raise the resonance velocity and enhance the wheel-rail contact force at resonance speed.

Keywords: Dynamic Response, Slab Track, Resonance, Stiffness, Bearing, Multibody.

Number: 10.14704/nq.2022.20.7.NQ33218 **Neuro Quantology 2022; 20(7):1741-1748**

Introduction: The slab track systems are widely used in metro and high speed railway transport in worldwide. Basically in slab track system the sleepers are replaced or combined with slabs. Slab track consists of the elements of rails, base plates, fasteners, rail pads and concrete (and/or reinforced concrete), and the hydraulically bonded layer (HBL). Due to the higher stiffness of the slab track, the dynamic interaction between the vehicle and the track becomes more obvious. Violent track vibration will limit service life and destroy the track. Therefore, lots of research work have been attached to analysis of dynamic response of railway slab track system in recent years. Kuo et al.[1] analyzed effects of slab bearing on track responses by coupled equilibrium equations of suspended wheels and floating slab track system. The correlation between wheel-rail resonance and train speed was also discussed. Wanming et al. [2] A fundamental model is established for analysing the train–track–bridge dynamic interactions, including the high-speed vehicle model, the ballasted and non-ballasted track models, the bridge model, as well as the wheel–rail interaction model and the track–bridge interaction model. Ling, L et al. [3] study of the dynamic interaction between urban railway vehicles and vibration-attenuating slab tracks Vehicle-track dynamic interactions are influenced by the features of vibration-attenuating slab tracks. Vehicles traveling on different vibration-attenuating slab tracks have notably varied dynamic reactions of wheel-rail forces. The track with the elastic-supporting-blocks has the best performance. J. Blanco et al.[4] The dynamic performance of a high speed ballasted track and three different types of slab tracks has been studied by numerical simulations in the time domain of a vehicle running on a straight section of track at high speed. The wheel-rail contact forces have a lower degree of fluctuations in the ballasted track and in the STEDEF track, than in the other two types of slab tracks studied. It might make sense to consider the distinct feature of rail supports in the analysis of wheel-rail resonance for the current train speeds. This is because, despite the low train running velocity, resonance in the track structure causes damage to rail and vehicle shaking. Few studies have been done to take resonance mechanism and slab track system circumstances into consideration because of the intricacy of track structure vibration. The dynamic response of the railroad's slab track systems to vehicle motion is the main topic of this research. The interaction between the vehicle and the slab track may be simulated at any speed using the moving mass on the slab and the vehicle-slab track coupling model presented in this study. By Using mathematical derivations and numerical simulations, the resonance mechanism and conditions of the slab track system are examined. The impact of rail pad stiffness and slab bearing stiffness on resonant frequency phenomena of the track system is examined in a numerical case study.

Vehicle Model for Analysis: A car body and two bogies comprise the vehicle subsystem. Vertical and pitch motion are allocated to the automobile body, front and rear bogies, respectively. The vehicle unit is then represented as a multi-body system with six degrees of freedom (DOFs) that runs at a constant velocity on the track, as shown in Fig. 1

The dynamic equation of motion of vehicle subsystem can be derived by Principal of the D' Alembert's and it can be expressed as:

$$M_v \ddot{a} + C_v \dot{a} + K_v a = Q_v \quad (1)$$

Where $\ddot{a}$, $\dot{a}$ *and* a *represent the acceleration,* velocity and displacement vectors of vehicle subsystem and the displacement vector can be expresses as:

$$a_v = [z_{cb} \quad \phi_{cb} \quad z_{fb} \quad \phi_{fb} \quad z_{rb} \quad \phi_{rb}] \quad (2)$$

Where, z_{cb} ,z_{fb} ,z_{rb} are represent the displacement on vertical direction of car body, the front and rear bogies of vehicle subsystem respectively and ϕ_{cb}, ϕ_{fb} , ϕ_{rb} are represent the pitching rotation of the car body, the front bogies and rear bogies respectively.

Mass matrix of vehicle subsystem , M_v can be written as:

$$M_v = Diag[m_{cb} \quad j_{cb} \quad m_{fb} \quad j_{fb} \quad m_{rb} \quad j_{rb}]; \quad (3)$$

Where m_{cb} ,m_{fb} &m_{rb} denote the mass of car body, front bogie & rear bogie respectively and j_{cb} , j_{fb} & j_{rb} are the pitch inertia of the car body, front bogie & rear bogie respectively.

Stiffness matrix of vehicle subsystem, K_v can be expressed as:

Where k_{sz}, k_{pz} are the vertical stiffness of secondary and primary suspension system and c_{sz}, c_{pz} are the vertical damping coefficient of secondary and primary suspension system, L_c is the half of the distance between center of slab track and L_b is the half of the distance of wheelbase.

$$K_v = \begin{bmatrix} 2k_{sz} & 0 & -k_{sz} & 0 & -k_{sz} & 0 \\ 0 & 2k_{sz} L_c^2 & k_{sz}L_c & 0 & k_{sz}L_c & 0 \\ -k_{sz} & k_{sz}L_c & k_{sz}+2k_{pz} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2L_b^2 k_{pz} & 0 & 0 \\ -k_{sz} & k_{sz}L_c & 0 & 0 & k_{sz}+2k_{pz} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2L_b^2 k_{pz} \end{bmatrix}$$

Damping matrix of vehicle subsystem C_v can be expressed as;

$$C_v = \begin{bmatrix} 2c_{sz} & 0 & -c_{sz} & 0 & -c_{sz} & 0 \\ 0 & 2c_{sz} L_c^2 & c_{sz}L_c & 0 & c_{sz}L_c & 0 \\ -c_{sz} & c_{sz}L_c & c_{sz}+2c_{pz} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2L_b^2 c_{pz} & 0 & 0 \\ -c_{sz} & c_{sz}L_c & 0 & 0 & c_{sz}+2c_{pz} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2L_b^2 c_{pz} \end{bmatrix}$$

The vector of loads acting on the vehicle is denoted byQ_v, can be represent as;

$$Q_v = [-m_{cb}g \quad 0 \quad m_{fb}g + Q_1 \quad -Q_2 \quad -m_{rb}g + Q_3 \quad -Q_4]$$

$Q_1 = k_{pz}\{z_{w1} + z_{w2}\}$; $Q_2 = L_b k_{pz}\{z_{w1} - z_{w2}\}$

$Q_3 = k_{pz}\{z_{w3} + z_{w4}\}$; $Q_4 = L_b k_{pz}\{z_{w1} + z_{w2}\}$

z_{wi}= the vertical displacement of the ith wheel.

Slab Track model: The dynamic behavior of the rail and slab is analyzed in the vertical plane, but deformations in the axial direction of the rail and slab are ignored. A uniform Bernoulli-Euler beam is used to model both the rail and the slab. The rail is characterized as a beam resting on slabs with discrete continuous elastic supports, and the slabs are characterized as beams with free ends on a continuous supporting foundation.

Finite element method is considered for obtained slab track model and can be written as:

$$M_t\ddot{z} + C_t\dot{z} + K_t z = F_t \quad (5)$$

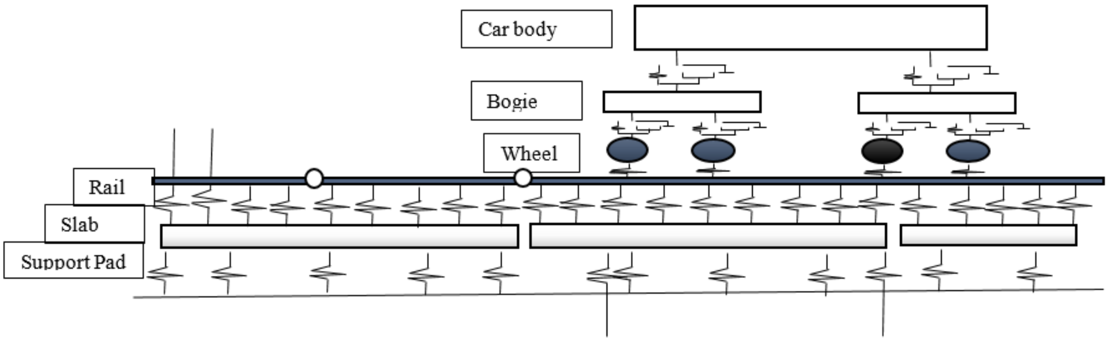


Fig.1 vehicle – slab track model

Where z is the displacement vector of beam element nodal, and can be written as:

$$z = \sum_{i=1}^{NE} z_i^e \quad (6)$$

$$z_i^e = \{ y_{2i-1} \quad \phi_{2i} \quad y_{2i+1} \quad \phi_{2i+2} \} \quad (7)$$

where z_i^e = ith element nodal displacement vector, NE= number of elements , y_{2i-1} & y_{2i+1} are the deflection in the vertical direction of the $(2i)^{th}$ & $(2i+2)^{th}$ element nodal, ϕ_{2i} , ϕ_{2i+2} are the rotation of the $(2i)^{th}$ & $(2i+2)^{th}$ element nodal.

The displacement vector can also be written in terms of shape functions [N] and nodal displacement [z] of beam element:

$$y(x,t) = [N]\{z_i^e\} \quad (8)$$

N is the shape function of beam, and the shape function can be written as

$$N_1 = 1 - 3\left(\frac{x^2}{L^2}\right) + 2\left(\frac{x^3}{L^3}\right) \quad ; \quad N_2 = -x + \left(\frac{2x^2}{L}\right) - \left(\frac{x^3}{L^2}\right) \quad (9)$$

$$N_3 = 3\left(\frac{x^2}{L^2}\right) - 2\left(\frac{x^3}{L^3}\right) \quad ; \quad N_4 = \left(\frac{x^2}{L}\right) - \left(\frac{x^3}{L^2}\right)$$

Where x is the local coordinate measured from left node of the element of beam and It should be observed that shape function includes only spatial parameters and $\{z_i^e\}$ that is time dependent. The derivative of the function y(x, t) can be represented as:

$$\frac{\partial y(x,t)}{\partial t} = [N]\{\dot{z}_i^e\} \quad ; \quad \frac{\partial^2 y(x,t)}{\partial^2 t} = [N]\{\ddot{z}_i^e\} \quad ; \quad \frac{\partial^2 y(x,t)}{\partial^2 x} = [N'']\{\dot{z}_i^e\}, \quad (10)$$

$$\frac{\partial y(x,t)}{\partial x} = [N']\{z_i^e\} \quad ; \quad \frac{\partial^2 y(x,t)}{\partial t \partial x} = [N']\{\dot{z}_i^e\},$$

M_t is the mass matrix of slab track, can be expressed as:

$$M_t = \Sigma_{i=1}^{n} m_r^e + \Sigma_{i=1}^{n} m_s^e \quad (11)$$

$$m_r^e = \frac{m_r L}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -2L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -13L^2 & -22L & 4L^2 \end{bmatrix},$$

$$m_s^e = \frac{m_s L}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -2L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -13L^2 & -22L & 4L^2 \end{bmatrix}$$

where: m_r^e is the mass matrix of rail element and m_s^e is the mass matrix of slab element. L is the length of element, m_r is the mass of rail per unit length, m_s is the mass of slab track per unit length.

$$K_r = \sum_{i=1}^{n} k_r^e + \sum_{i=1}^{n} k_s^e \quad (12)$$

k_r^e and k_s^e are the stiffness matrix of rail element and slab track respectively, and may be expressed as;

$$k_r^e = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}; \quad k_s^e = \frac{k_f L}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -2L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -13L^2 & -22L & 4L^2 \end{bmatrix}$$

Where, E is the modulus of elasticity of rail, I is the moment of inertia of rail and k_f is the elastic coefficient of continuous foundation.

F_t is the total load vector acting on rail , can be expressed as :

$$F_t = \sum_{i=1}^{nw} R_p^e \quad (13)$$

$$R_p^e = N^T F_{wi}$$

F_{wi} *is the force between rail and wheel.*

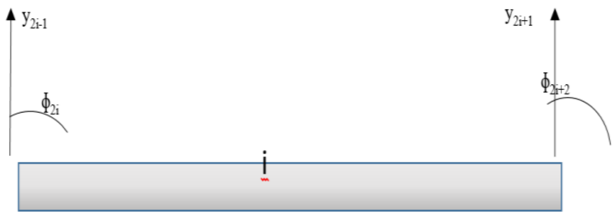


Fig 2. Element of Beam

Wheel and Rail Contact relation:

Although each wheel is supposed to be in regular contact with the rail beam, the wheel's motion can be represented as;

$$z_{wi} = N\{z_i^e\}$$
$$\dot{z}_{wi} = vN'\{z_i^e\} + [N]\{\dot{z}_i^e\}, \quad (14)$$
$$\ddot{z}_{wi} = \alpha N'\{z_i^e\} + 2vN'\{z_i^e\} + v^2N''\{z_i^e\} + [N]\{\ddot{z}_i^e\},$$

$z_i^e = \{ y_{2i-1} \quad \phi_{2i} \quad y_{2i+1} \quad \phi_{2i+2}\}$

Where, v is represent vehicle velocity in longitudinal direction and α is represent the acceleration in the longitudinal direction.

Force occur due to motion of wheel on rail, can be written as:

$$F_{w1} = k_{pz}(-z_{fb} + L_t\theta_{fb} + z_{w1}) + m_w\ddot{z}_{w1} + m_w g \quad (15)$$
$$F_{w2} = k_{pz}(-z_{fb} - L_t\theta_{fb} + z_{w2}) + m_w\ddot{z}_{w2} + m_w$$
$$F_{w3} = k_{pz}(-z_{rb} + L_t\theta_{rb} + z_{w3}) + m_w\ddot{z}_{w3} + m_w g$$
$$F_{w4} = k_{pz}(-z_{rb} - L_t\theta_{rb} + z_{w4}) + m_w\ddot{z}_{w4} + m_w g$$

The total stiffness matrix, total mass matrix, and total damping matrix for the vehicle and track coupling system can be formed based on the relationship between wheel and rail. The Newmark- integration method, which is applied in this work, can be used to solve the vehicle and track system equations.

2.2 Resonant velocity of loaded track

The frequency f_t of loaded slab track system on the elastic foundation of uniform stiffness can be approximately derive by

$$f_t = \frac{1}{2\pi}\sqrt{\frac{k_{tr}}{m_{tr}+m_w}} \quad (16)$$

$$k_{tr} = 2\sqrt[4]{k_f^3 \times 4EI} \quad ; \quad m_{tr} = 3m \times \sqrt[3]{\frac{EI}{k_{tr}}}$$

Where, k_{tr} & m_{tr} are effective stiffness & mass of elastic foundation, k_f is the equivalent stiffness per unit length of the foundation.

Coupled vehicle-slab track system moving with a constant velocity v travelling on the track which is supported by discrete rail- pads of equal intervals, excitation frequency f_{extn} of the wheel load track system due to the discrete rail-pads is;

$$f_{extn} = \frac{v}{l_e} \quad (17)$$

Where

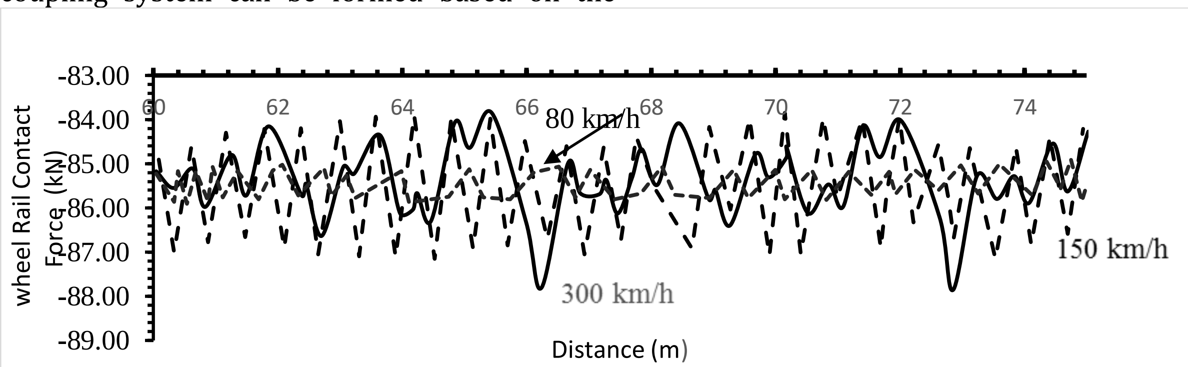
l_e is the effective distance between two adjacent rail pads.

Resonance can be excited between the bogies and the rails when the excitation frequency equals the loaded track frequency. The resonant speed can be calculated using the following equation:

$$v_r = \frac{l_e}{2\pi}\sqrt{\frac{k_{tr}}{m_{tr}+m_w}} \quad (18)$$

Analysis of slab track system

Contact Force between Rail and Wheel at different speeds: The slab bearing stiffness is fixed to 6.5 x 10^7 N/m, the rail pad stiffness is fixed to 6 X10^7 N/m, and the contact forces between wheel and rail are evaluated at speeds of 300 km/h, 150 km/h, and 80 km/h, which is shown in Fig. 3



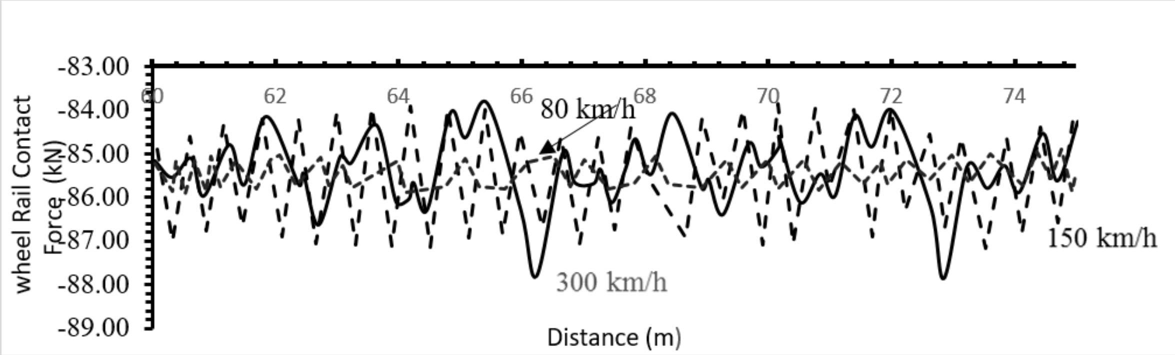
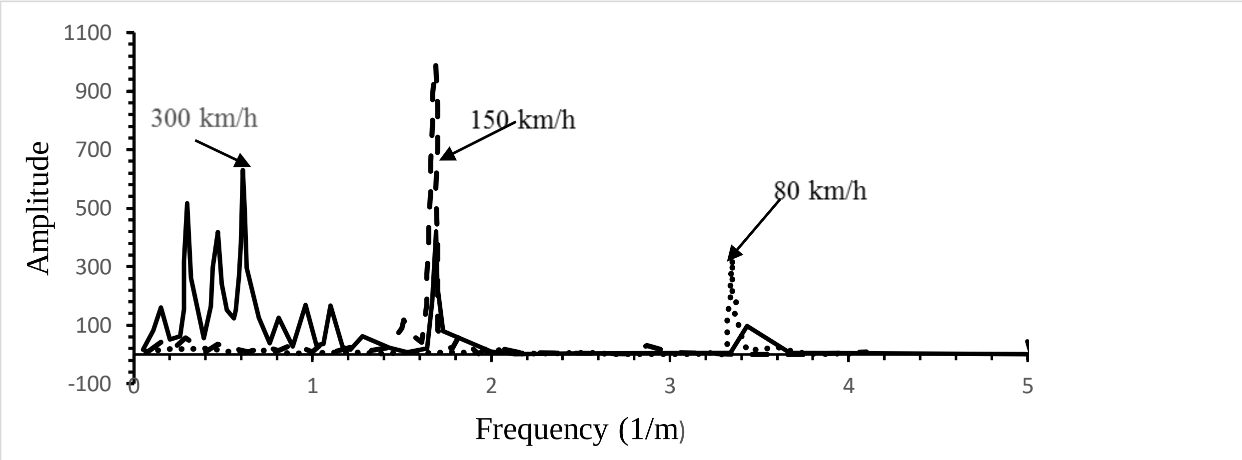


Fig 3. Wheel rail Contact force at different speed



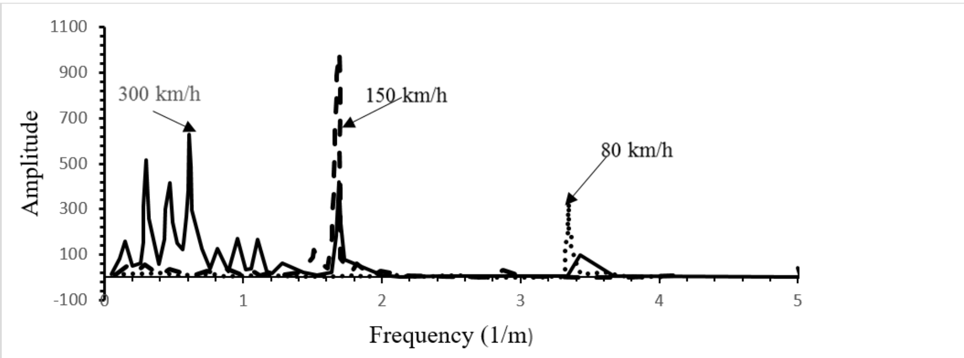


Fig 4. The Frequency Response of contact force at train different speeds

Assessment the effect of rail pad stiffness: The bearing stiffness of concrete slab in this assessment is fixed to k_s is equal to the 7.0 X 10^8 N/m and the bearing stiffness of rail pad system are vary as k_{pad}= 6.0x 10^7 N/m, k_{pad} = 7.4x 10^7 N/m and k_{pad} = 8.5 x 10^7 N/m. by varying the bearing stiffness of rail pad the resonance speeds generated by the 0.65 m spacing of rail pad can be estimated under as shown in Table 3. It can be seen that as the rail pad stiffness is increased, the resonance speed increases.

Table1: Resonance speeds of slab track system under various rail pad stiffness

Stiffness of Rail Pad k_{pad}(N/m)	Resonant speed (km/h)				
	n = 1	n =2	n = 3	n = 4	n =5
6.0x 10^7	120	240	350	475	532
7.4x 10^7	135	265	385	515	634
8.5 x 10^7	146	286	423	576	683

Impact factor: The impact factor is required in design to compensate the magnification effect on the vehicle and track reaction. The impact factor is defined as:

$$I_f = \frac{Q_{dy}(x) - Q_{st}(x)}{Q_{st}(x)}$$

Where $Q_{st}(x)$ &$Q_{dy}(x)$ are represent the maximal dynamic and static responses of slab track system by the actions of moving loads.

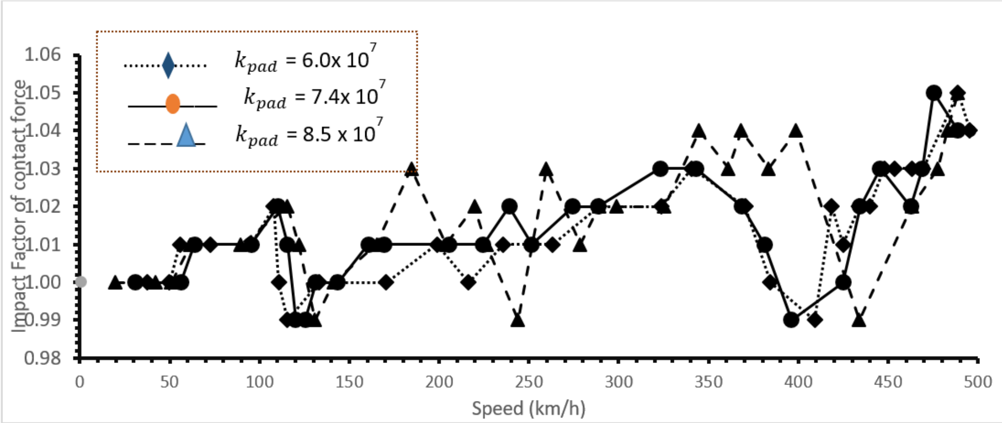
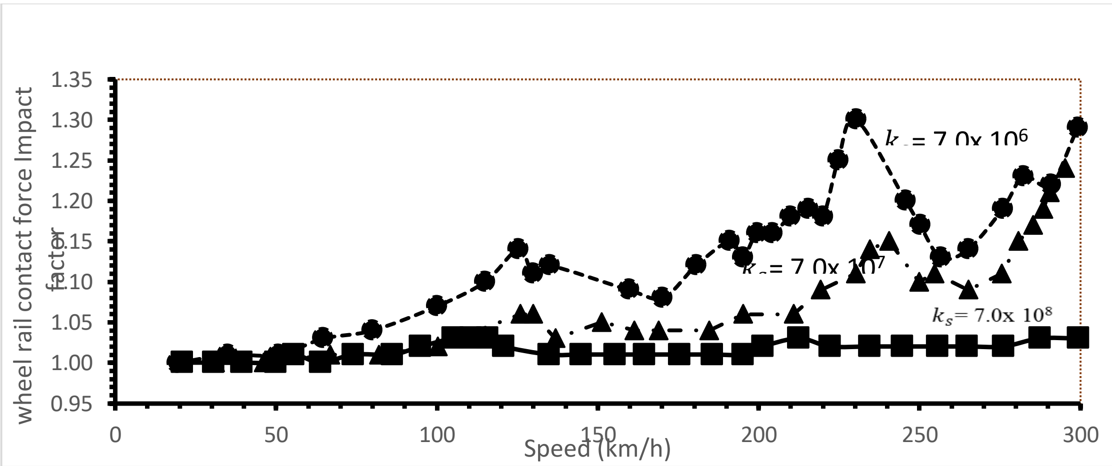


Fig 5.Impact factor for wheel rail contact force at different rail pad stiffness

Fig 4 & 5 demons trades the impact factor of the wheel rail contact force for various rail pad stiffness. The wheel-rail resonance appears to be really worse and the resonance frequency appears to be intensified as the rail pad stiffness increases. The first-order resonance speed is correlated with the peak forces between the wheel and rail. With an increase in rail pad stiffness, numerical simulations show a modest rise in the frequency speed of contact force.

Assessment the effect of bearing stiffness of concrete slab

The bearing stiffness of rail pad in this assessment is fixed to k_{pad}= 6.0x 10^7 N/m and the bearing stiffness of concrete slab system are vary as $k_s = 7.0x$ 108 N/m, k_s = 7.0x 10^7 N/m and k_s = 7.0 x 10^6 N/m. The frequency spectrum of contact force between rail and wheel set of track at the velocity of 150 km/h and 300km/h is determine. The vehicle is moving at the velocity of 150km/h on track, & decrease in bearing stiffness of slab, wheel rail contact force induced by the slab span of 6 m is more noticeable at 0.17 Hz frequency. If the bearing stiffness of slab is reduced at a speed of vehicle 300 km/h, the contact force between wheel- rails resulting from slab bending is quickly induced at 0–1 Hz, and there are different force frequency components. When the slab bearing stiffness is high, the force frequency component of 1.67 Hz (0.6 m) that is induced by the rail pad spacing will account for a significant amount of the force of any kind speeds.



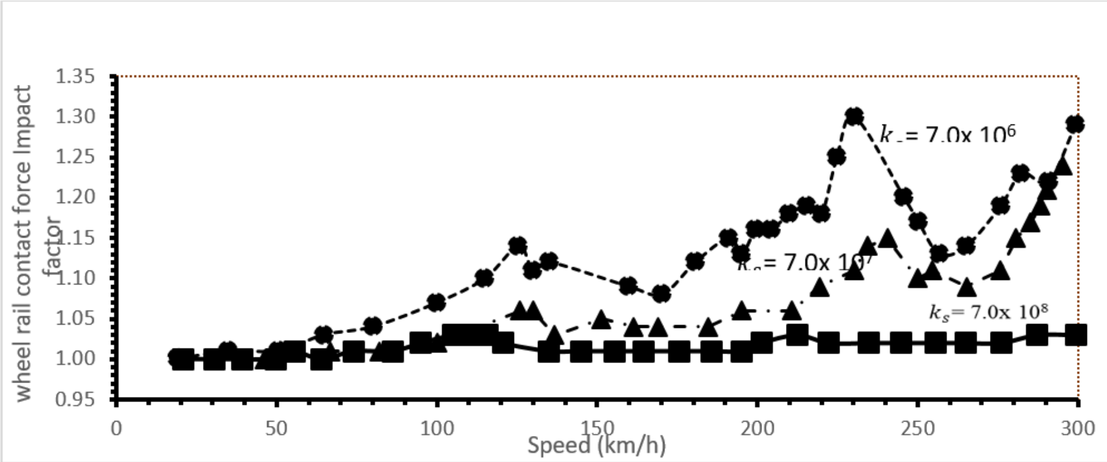


Fig 6.Impact factor for wheel rail contact force at different slab bearing stiffness

Fig 6. Demons trades the impact factor of the wheel rail contact force for various bearing stiffness of concrete slab.

The wheel-rail force changes sharply and exhibits a significant accelerating trend because when bearing stiffness of concrete slab decreases. Wheel-rail force resonance peaks are clearly visible. The peak impact response at resonance speed is more visible the lower the slab bearing stiffness is. The resonant peak doesn't match the theoretical prediction in Table 1 because when speed and slab bearing stiffness increase, the frequency component caused by slab bending will be higher and the frequency component generated by rail pad spacing tends to be smaller. Overall, the decreased slab bearing stiffness will increase dynamic responsiveness, alter the frequency component of the wheel-rail force, and make it more challenging to estimate rail resonance.

Conclusions: A beam model of slab track with discrete rail pads was developed for dynamic response analysis. The entire model of track is categorized as in two subsystems for analysis: the track subsystem is viewed as elastic beam model, and the vehicle subsystem is viewed as a rolling stock unit with primary suspension systems and secondary suspension systems. Using the finite element method, the dynamic frequency response of railway slab track system under moving vehicle system is evaluated. The wheel rail contact force response are examine by using theoretical derivations and numerical simulations. The following conclusions are based from the dynamic response analysis of the vehicle and slab track nonlinear coupling system:

1) The slab track has a low vibration attenuation, making it easier to create the peak resonance. The response based on by rail pad spacing is more pronounced the higher the slab bearing stiffness is. About 100 km/h is the resonance speed. Wheel-rail wear may simply get worse in practice if the track damping is too low since resonance will be considerably more intense.
2) Increasing the stiffness of rail pad will raise the resonance velocity and enhance the wheel-rail contact force at resonance speed.
3) The wheel-rail force frequency components induced by slab deflection and vibration are easily inspired and there are multiple force frequency components as the speed increase and slab bearing stiffness decrease. The significance of dynamic response at resonance speeds increases, and the resonance peaks seem to be more frequent.

References

[1] Kuo C., Huang C., Chen Y**.** Vibration characteristics of floating slab track. Journal of Sound and Vibration, Vol. 317, Issues 3-5, 2008, p. 1017-1034

[2] Zhai, W., Xia, H., Cai, C., Gao, M., Li, X., Guo, X. & Wang, K. (2013). High-speed train–track–bridge dynamic interactions–Part I: a theoretical model and numerical simulation. *International Journal of Rail Transportation*, *1*(1-2), 3-24.

[3] Ling, L., Jiang, P., Wang, K., & Zhai, W. (2020). Dynamic interaction between rail vehicles and vibration-attenuating slab

tracks. *Construction and Building Materials*, *258*, 119545

[4] Blanco-Lorenzo, J., Santamaria, J., Vadillo, E. G., & Oyarzabal, O. (2011). Dynamic comparison of different types of slab track and ballasted track using a flexible track model. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, *225*(6), 574-592.

[5] Luo, Y., Yin, H., & Hua, C. (1996). The dynamic response of railway ballast to the action of trains moving at different speeds. *Proceedings of the institution of mechanical engineers, Part F: Journal of rail and rapid transit*, *210*(2), 95-101.

[6] Dong, R. G., Sankar, S., & Dukkipati, R. V. (1994). A finite element model of railway track and its application to the wheel flat problem. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, *208*(1), 61-72.