STUDY OF REDUNDANT SYSTEM WITH FAILURE AND REPAIR TIME

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Abstract:
In this paper, we have studied a two-unit cold standby system under failure and repair time. There is a single repairman who visits the system immediately after failure. Here, the failure rate follows exponential distribution and the repair rate follows general distribution.There are two types of failure of primary (operative) unit, i.e., partial and complete failure whereas the standby unit undergoes only under complete failure. Various reliability measures like mean time to system failure (MTSF), steady-state availability, and busy-period are derived using semi-Markov process. The comparative analysis of various reliability measure has been drawn graphically by taking some arbitrary values.

Keywords: Failed unit, repaired unit, repairman, MTSF, Availability, Busy-period analysis.
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Introduction

Reliability was the primary focus for statisticians and engineers in the early 1950s, following World War II, when testing electronic devices and missiles was the major focus. Studies carried out by scientists discovered new measurements and the system's dependability. Rizwan and Taneja (2001) performed profit evaluation with partial/complete modes of failures. In many practical situations, for example, dim light concept, partial failure concept is used.Kumar and Vashishth (2011) aimed to investigate the dependability of a two-unit cold standby redundant system that experiences common cause failures and the additional variable approach was used to obtain the reliability expression along with the numerical findings for a specific situation. Hirata et al. in 2019 studiedthat adding redundancy to an architecture is a well-known and useful way to increase reliability in which a two-component standby redundant system with priority was taken into consideration in some earlier studies, and the system's dependability function, variance of failure time (VOFT), and mean time to failure (MTTF) were assessed using the limited stochastic information available for each component. Specifically, the structure for the digital SC twin is introduced.The research investigation by Rykov et al. (2020) delivers a reliability analysis of a cold, double redundant renewable energy system. The Laplace-Stieltjes transform of two primary reliability metrics, such as the failure-to-first and failure-to-interval distributions and the appropriate mean timings along with the time-dependent and time-stationary state probabilitieswere computed using the transformations.In a study from 2023, Gao Shan et al. examined a redundant series system with one repairman and one backup unit for every primary unit. The repairman has a postponed vacation if the system has no failed units. For such a system, we first extract the system's steady-state distribution and derive certain reliability and queueing metrics by constructing the system's balancing equations and solving a second-order linear non-homogeneous difference equation with variable coefficients. Additionally, we do a thorough reliability study using the Markov renewal approach.

In this paper, we examine a redundant system with two identical units. The first unit experiences both partial and complete failure modes, whereas the second one experiences total failure. These systems are seen in real-world applications in the electronics sector. A repair person who is knowledgeable about two distinct repairs and the time it takes to do them as general repairs for failed units. A system breakdown happens when a unit is not in use. By considering the time distribution as exponential and others as general, relevant measurements that indicate the system's efficacy are produced. "Steady-State Availability of the System," "Expected Busy Period of Repairman," and "Mean (Average) Time to System Failure" for both partially and fully failing units. Visual graphical investigations are used to get findings regarding MTSF and system profit. First isoperative, other is cold standby.

1. The first has 3 modes, that is, operative, partial and complete failure.
2. Secondhas only one failure type, complete failure.
3. Distribution time of failure is exponential, although arbitrary for repair time distribution.
4. Always there is availability of repairman.
5. After repair, unit functions perfectly.
6. Breakdown of the system occurs if both of the units are under repair.
7. A unit works properly after every repair.
8. Each of the random variable is independent.

Notations used in System

α: Rate of partial failure of first unit

β: Rate of total failure offirst unit

γ : Rate of total failure of second unit

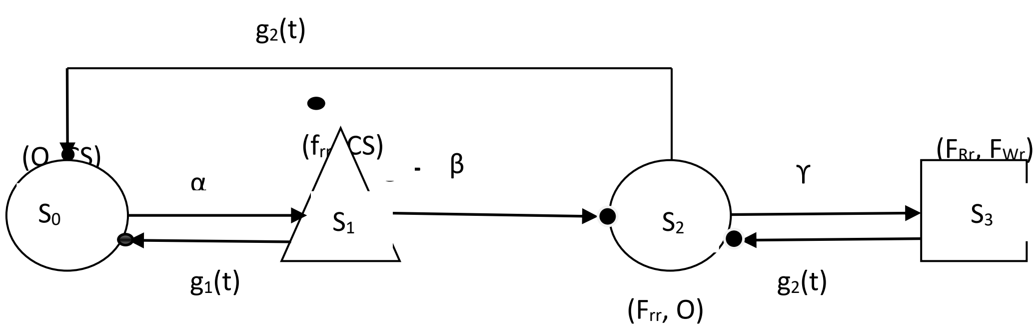
$g_1(t)$, $G_1(t)$: p.d.f. and c.d.f. of repair time for first unit

$g_2(t)$, $G_2(t)$: p. d. f. and c. d. f. of repair time for second unit

$P_{i,j}^{(k)}$: Probability of system goes from regenerative state S_i to S_j via S_k non-regenerative state

μ_i : Average sojourn time in S_i

Transition Diagram



Transition Probabilities

P_{ij}: Probability of Transition from S_i to S_j and direct transition probabilities, S_i to S_j are given by

$Q_{ij}(t) = \int_0^t f(u)du$ or $\frac{dQ_{ij}(t)}{dt} = \int_0^t \frac{df(t)}{dt}dt \Rightarrow q_{ij}(t) = f(t)$

$$Q_{0,1}(t) = \int_0^t \alpha e^{-(\alpha)u}\, du$$

$$Q_{1,0}(t) = \int_0^t e^{-(\beta)u} g_1(u)\, du$$

$$Q_{1,2}(t) = \int_0^t \beta\, e^{-(\beta)u}\overline{G_1(u)}\, du$$

$$Q_{2,0}(t) = \int_0^t e^{-(\gamma)u}\, g_2(u)\, du$$

$$Q_{2,3}(t) = \int_0^t \gamma e^{-(\gamma)u}\, \overline{G_2(u)}\, du \qquad (1-5)$$

The Steady State Transition Probabilities are given by

$$q_{ij}(t) = \frac{d}{dt}Q_{ij}(t)$$

$$P_{ij} = \lim_{s\to 0} q_{ij}^*(s) = \lim_{s\to 0}\int_0^\infty e^{-st}q_{ij}(t)dt = \lim_{t\to\infty} Q_{ij}(t)$$

$q_{0,1}(t) = \alpha e^{-\alpha t}$

$P_{0,1} = 1$

$q_{1,0}(t) = e^{-\beta t}g_1(t)$

$P_{1,0} = g_1^*(\beta)$

$q_{1,2}(t) = \beta e^{-\beta t}\overline{G_1(t)}$

$P_{1,2} = 1 - g_1^*(\beta)$

$q_{2,0}(t) = e^{-\gamma t}g_2(t)$

$P_{2,0} = g_2^*(\gamma)$

$q_{2,3}(t) = \gamma e^{-\gamma t}\overline{G_2(t)}$

$P_{2,3} = 1 - g_2^*(\gamma)$

$q_{2,2}^{(3)} = (1 - e^{-\gamma t})\, g_2^*(\gamma)$

$P_{2,2}^{(3)} = 1 - g_2^*(\gamma) \qquad (6-17)$

From above, we get

$P_{0,1} = 1, P_{2,0} + P_{2,3} = 1, P_{1,0} + P_{1,2} = 1, P_{2,0} + P_{2,2}^{(3)} = 1 \qquad (18-21)$

The unconditional mean time taken by the system to go from regenerative state j, it is measured from span of entrance into the state i.

$$m_{ij} = \int_0^\infty t\, dQ_{ij}(t) = \int_0^\infty t\, q_{ij}(t)dt = -Q_{ij}^{*'}(s)$$

Thus,

$m_{0,1} = \mu_0$,$m_{1,0} + m_{1.2} = \mu_1, m_{2,0} + m_{2.3} = \mu_2$,$m_{2,0} + m_{2,2}^{(3)} = K'$ (22-26)

where $K' = -g_2^{*'}(0)$

Average Time to Failure

Recursive relations,$\phi_i(t)$given

$\phi_0(t) = Q_{01}(t) \circledS \phi_1(t)$

$\phi_1(t) = Q_{10}(t) \circledS \phi_0(t) + Q_{12}(t) \circledS \phi_2(t)$

$\phi_2(t) = Q_{20}(t) \circledS \phi_0(t)$ (27-29)

Taking Laplace-Stieltjes Transforms (L.S.T.) and solving for $\phi_0^{**}(s)$,

$\phi_0^{**}(s)=\frac{N(s)}{D(s)}$

$N(s) = q_{01}^*(s)q_{12}^*(s)q_{23}^*(s)$

$$D(s) = 1 - q_{01}^*(s)q_{12}^*(s)q_{20}^*(s) + q_{01}^*(s)q_{10}^*(s)$$

The mean time to system failure when the system starts from state 0 is

$$MTSF = \lim_{s\to 0} \frac{1-\phi_0{}^{**}(s)}{s} = \frac{N}{D} \quad (30)$$

where $N = \mu_0 + \mu_1 P_{01} + \mu_2 P_{01}P_{12},$

and $D = P_{12}P_{23}.$

Availability Analysis

Let $A_i(t)$ be the probability that the system is in up-state at instant t given that the system entered regenerative state i at $t = 0$. The availability $A_i(t)$ seems to satisfy the following recursive relations:

$A_0(t) = M_0(t) + q_{01}(t)©A_1(t)$

$A_1(t) = M_1(t) + q_{10}(t)©A_0(t) + q_{12}(t)©A_2(t)$

$A_2(t) = M_2(t) + q_{20}(t)©A_0(t) + q_{22}^{(3)}(t)©A_2(t)$ (31-33)

where$M_0(t) = e^{-\alpha t}$, $M_1(t) = e^{-\beta t}\overline{G_1}(t)$, $M_2(t) = e^{-\gamma t}\overline{G_2}(t)$.

Taking the Laplace Transform of these equations, when the system starts from S_0, solving these, we get

$A_0^*(s)=\frac{N_1(s)}{D_1(s)}$

where

$$N_1(s) = M_0^*(s)\left(1 - q_{22}^{(3)*}(s)\right) + q_{01}^*(s)\left(1 - q_{22}^{(3)*}(s)\right)M_1^*(s) + q_{01}^*(s)q_{12}^*(s)M_2^*(s)$$

$$D_1(s) = \left(1 - q_{22}^{(3)*}(s)\right) - q_{01}^*(s)q_{10}^*(s)\left(1 + q_{22}^{(3)*}(s)\right) + q_{01}^*(s)q_{12}^*(s)q_{20}^*(s)$$

In steady state, the availability of the system is given by

$$A_0 = \lim_{s\to 0} s\, A_0^*(s) = \frac{N_1(s)}{D_1(s)} \quad (34)$$

where$N_1 = \mu_0 P_{01}P_{20} + \mu_1 P_{01}P_{20} + \mu_2 P_{12},$

and$D_1 = \mu_0 P_{20} + P_{20}K + P_{12}K'.$

where $K = -g_1^{*'}(0), K' = -g_2^{*'}(0)$

Busy Period of Repairman

Let $B_i(t)$be period for which repairman is busy starting from regenerative state S_i at t = 0"
$B_0(t) = q_{01}(t)©B_1(t)$
$B_1(t) = W_1(t) + q_{10}(t)©B_0(t) + q_{12}(t)©B_2(t)$
$B_2(t) = W_2(t) + q_{20}(t)©B_0(t) + q_{22}^{(3)}(t)©B_2(t)$ (35-37)
where $W_1(t) = \overline{G_1}(t), W_2(t) = \overline{G_2}(t)$.

Taking Laplace Transforms and solving for $B_0^*(s)$,
$B_0^*(s)=\frac{N_2(s)}{D_1(s)}$
where $N_2(s) = q_{01}^*(s)(1 - q_{22}^{(3)*}(s))W_1^*(s) + q_{01}^*(s)q_{12}^*(s)$
$D_1(s)$is defined above.

In steady state, the total fraction of time which the system is under repair of the repairman,
$B_0 = \lim_{s\to 0} s\, B_0^*(s) = \frac{N_2(s)}{D_1(s)}$ (38)
where $N_1 = P_{20}K + P_{12}K'$,
andD_1 is already defined above. 5139

Numerical Study and Graphical Analysis
Considering the following numerical values:

$$\boldsymbol{\alpha = 0.005}, \quad \boldsymbol{\beta = 0.005, \gamma = 0.005, \delta = 0.04}, \quad \boldsymbol{\lambda = 0.03}$$

Measures that give effectiveness of system:
Mean time to system failure (MTSF)(T_0) = $\mathbf{290}$
Steady state availability (A_0) = $\mathbf{0.8210}$
Busy Period analysis of Repairman $(B_0) = \mathbf{0.1823}$
Expected Number of Visits by Repairman $(V_0) = \mathbf{0.0021}$
Graphs plotted for MTSF, Availability, Profit for various values of failure, repair rate. Fig—1 is graph plotted between MTSF(T_0) and failure rate (α) for fixed repair rate (δ). Graph reveals that MTSF decreases as values of (α) increases.

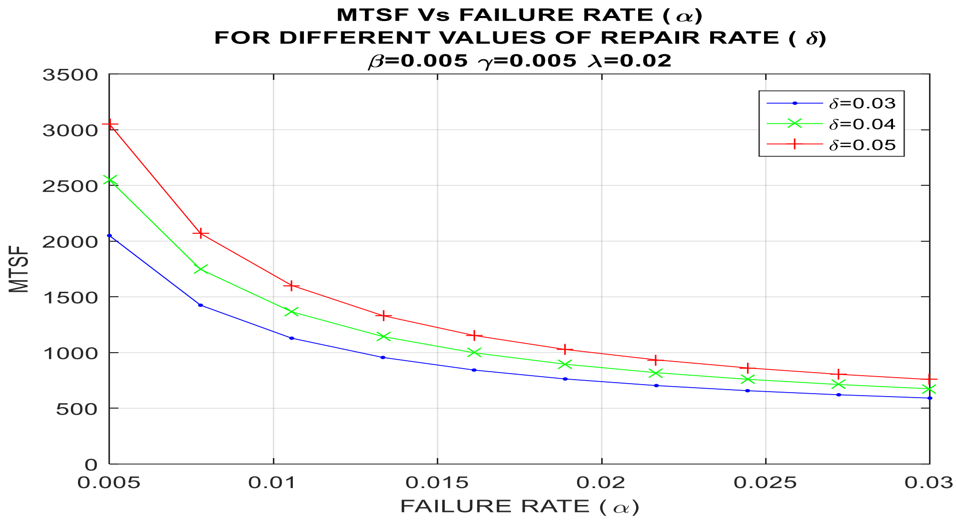


Fig. −1. MTSF versus Failure Rate

In Fig − 2, graph is plotted between Availability (A_0) and failure rate (α) for fixed repair rate (δ). Graph reveals, (A_0) decreases as (α) value decreases.

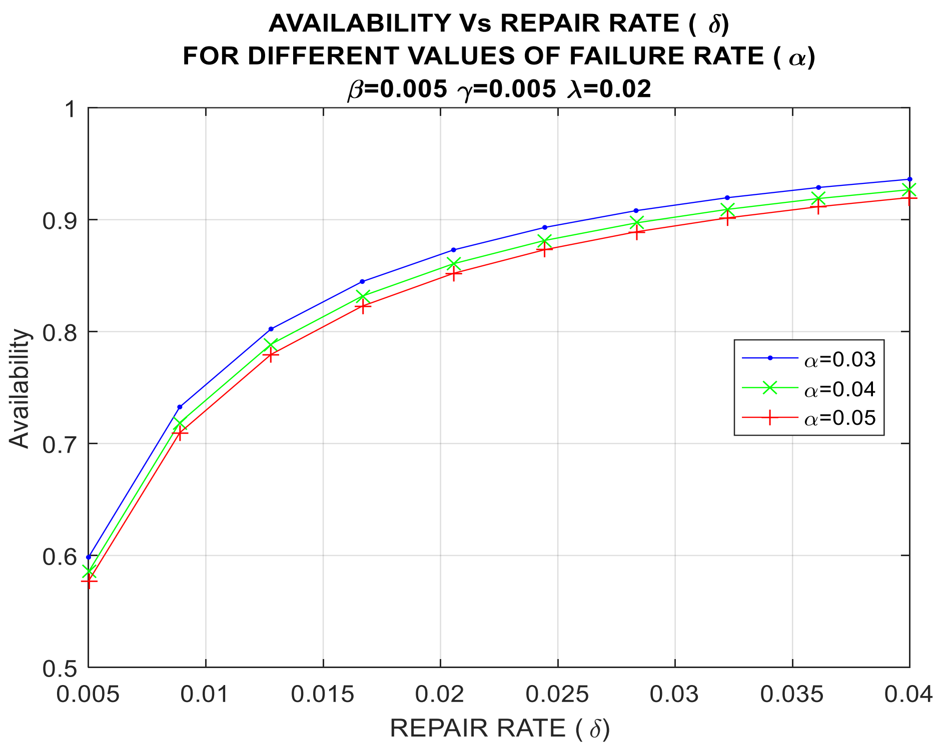


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Fig. − 2. Availability versus Repair Rate

Availability (A_0) and repair rate (δ). Graph is clearly depicting Availability (A_0) increases as we increase repair rate (δ), values of failure rate (α) are fixed.

Conclusion

From above graphs, it can be easily observed that if the value of failure rate increases, then there is decrease in the MTSF. Hence, smaller is rate of failure, higher is reliability, profit of system. This would help to decide how much revenue per unit uptime to have positive profit from system.

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